

ARTICLE

Open Access

Position-correlated biphoton wavefront sensing for quantum adaptive imaging

Yi Zheng^{1,2,3}, Zhao-Di Liu^{1,2,3}, Jian-Shun Tang^{1,2,3,4}, Jin-Shi Xu^{1,2,3,4}✉, Chuan-Feng Li^{1,2,3,4}✉ and Guang-Can Guo^{1,2,3,4}

Abstract

Quantum imaging with spatially entangled photons offers advantages such as enhanced spatial resolution, robustness against noise, and counterintuitive phenomena, while a biphoton spatial aberration generally degrades its performance. Biphoton aberration correction has been achieved by using classical beams to detect the aberration source or scanning the correction phase on biphotons if the source is unreachable. Here, a new method named position-correlated biphoton Shack–Hartmann wavefront sensing is introduced, where the phase pattern added on photon pairs with a strong position correlation is reconstructed from their position centroid distribution at the back focal plane of a microlens array. Experimentally, biphoton phase measurement and adaptive imaging against the disturbance of a plastic film are demonstrated. This single-shot method is a more direct and efficient approach toward quantum adaptive optics, suitable for integration into quantum microscopy, remote imaging, and communication.

Introduction

Entangled photons play a critical role in the development of quantum information and technology¹. Quantum imaging, which fully utilizes the spatial degree of freedom of photons, can achieve several nonclassical optical effects^{2,3}. Ghost imaging⁴ and quantum imaging with undetected photons⁵ are counterintuitive imaging methods. By joint probability distribution (JPD) measurement, spatially entangled photons can achieve a higher spatial resolution than the Rayleigh limit^{6–8}, which is beneficial in optical microscopy. Also, they can be distilled from the stray light^{9,10}. However, the atmospheric turbulence or flaws in optical instruments introduce phase aberrations and degrade the imaging performance in both classical and quantum imaging¹¹. Adaptive optics, including phase measurement and correction, is dedicated to overcoming this problem.

In classical optics, a famous phase measurement method is Shack–Hartmann wavefront sensing (SHWS)^{12–14}, which

uses a microlens array to focus the light inside each aperture. The local obliquity of light within an aperture, corresponding to the phase gradient, is mapped to the displacement of the spot at the microlens back focal plane. The measured gradient distribution is discretized according to the microlens width, which limits its spatial resolution. Then, the phase distribution is reconstructed by the zonal or modal method¹⁵ and corrected by a spatial light modulator (SLM) or a deformable mirror. There are also other types of wavefront sensors^{14,16,17} and even sensorless adaptive optical techniques. The basic idea of a notable sensorless one is that with an aberration, the focused spot after a Fourier lens increases in size and decreases in peak intensity, which serves as the criterion to perform feedback control of the correction phase till the sharp peak revives¹⁸.

In quantum optics, the spatial phase of entangled photon pairs (biphotons) can be measured by holography using a reference beam^{19,20} or the polarization entanglement^{21,22}, and the aberration cancellation of biphotons correlated in position by measuring the aberration source with classical lights has been demonstrated^{23–25} (in our article, the word “correlated” means the two photons are approximately at the same position, contrary to the term “anti-correlated,” and does not mean they can have an arbitrary nonseparable JPD). On the other hand, if a

Correspondence: Jin-Shi Xu (jsxu@ustc.edu.cn) or

Chuan-Feng Li (cfl@ustc.edu.cn)

¹Laboratory of Quantum Information, University of Science and Technology of China, 230026 Hefei, China

²Anhui Province Key Laboratory of Quantum Network, University of Science and Technology of China, 230026 Hefei, China

Full list of author information is available at the end of the article

© The Author(s) 2025



Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.



phase aberration from an unreachable source has already been added to position-correlated biphotons, a prominent measurement method has been demonstrated by Cameron et al.¹¹. In their protocol, inspired by the sensorless method¹⁸, the criterion is the peak value of the biphoton position sum-coordinate (or centroid, equivalently) marginal distribution at the Fourier plane. By scanning the coefficients of Zernike polynomials, the aberration can be eliminated. This indirect way requires multiple measurement steps, so the optimal group of coefficients may not be easily obtained by sequential scanning rather than using special algorithms. In this work, we introduce a single-shot method to realize this task, namely position-correlated biphoton SHWS (PCB-SHWS), which directly measures the gradient of phase added on position-correlated biphotons. Then, using biphotons from spontaneous parametric down-conversion (SPDC)^{26,27}, the phase measurement and adaptive imaging is experimentally demonstrated.

Results

Theoretical framework

The most fundamental idea of PCB-SHWS is from the Einstein–Podolsky–Rosen paper²⁸. Denoting the transverse position $\boldsymbol{\rho} = (x, y)$ and momentum $\mathbf{q} = (k_x, k_y)$, letting the biphoton field in front of a lens with the focal length f_{SH} have a constant intensity, a perfect position correlation, and the phase of an oblique plane wave $e^{i\mathbf{q}_0 \cdot \boldsymbol{\rho}}$ added on each photon, the wavefunction (joint amplitude)^{20,29,30} in the position space $\psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)e^{2i\mathbf{q}_0 \cdot \boldsymbol{\rho}_1}$, and their momenta are perfectly anti-correlated $\tilde{\psi}(\mathbf{q}_1, \mathbf{q}_2) = \delta(\mathbf{q}_1 + \mathbf{q}_2 - 2\mathbf{q}_0)$ with the anti-correlation center $\mathbf{q}_0$. Denoting their wavelength λ and wave number $k = 2\pi/\lambda$, the position wavefunction at the back focal plane of the lens is $\tilde{\psi}(f_{\text{SH}}\mathbf{q}_1/k, f_{\text{SH}}\mathbf{q}_2/k)$ with a paraboloid phase added [because $\psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is not the wavefunction at the front focal plane] which does not affect the JPD $\Gamma(\mathbf{q}_1, \mathbf{q}_2) = |\tilde{\psi}(\mathbf{q}_1, \mathbf{q}_2)|^2$ (for simplicity, we directly use $\mathbf{q}$ and ignore the normalization). So, after measuring the JPD, denoting the centroid $\mathbf{q}_c = (\mathbf{q}_1 + \mathbf{q}_2)/2$, summing JPD values of point pairs with the same centroid yields the biphoton centroid marginal distribution

$$\Gamma_c(\mathbf{q}_c) = \int d\mathbf{q}_1 \Gamma(\mathbf{q}_1, 2\mathbf{q}_c - \mathbf{q}_1) \propto \delta(\mathbf{q}_c - \mathbf{q}_0) \quad (1)$$

which is a sharp peak at $\mathbf{q}_0$. When photons at the whole microlens array arrive at its back focal plane, ignoring the momentum anti-correlation weakening caused by the finite microlens width $2a$, if the phase at each aperture can be approximated by an oblique plane wave and the dynamic range of SHWS $|q_{0x}|, |q_{0y}| < ka/f_{\text{SH}}$ is satisfied, photon pairs from different apertures have distinct centroids which are inside their own aperture, and the

whole centroid distribution is an array of sharp peaks, similar as the measured intensity distribution of classical SHWS. The gradient values of the whole phase pattern can be calculated from the peak positions relative to their centers.

Then, we consider the effects of the finite microlens width, a finite position correlation with the form $c(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$ whose width is far less than $2a$, and the added phase $\Phi(\boldsymbol{\rho})$, which is generally not a plane wave. The position wavefunction at an aperture S centered by $\mathbf{0}$ is

$$\psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = c(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)U(\boldsymbol{\rho}_1)U(\boldsymbol{\rho}_2) \quad (2)$$

where $U(\boldsymbol{\rho}) = e^{i\Phi(\boldsymbol{\rho})}\text{rect}[\boldsymbol{\rho}/(2a)]$ and the two-dimensional rectangular function $\text{rect}(\boldsymbol{\rho}) = 1$ when $|x| \leq 1/2, |y| \leq 1/2$ and 0 elsewhere. Denoting the Fourier transforms $\tilde{U}(\mathbf{q}) = \int d\boldsymbol{\rho} U(\boldsymbol{\rho})e^{-i\mathbf{q} \cdot \boldsymbol{\rho}}$ and $\tilde{c}(\mathbf{q}) = \int d\boldsymbol{\rho} c(\boldsymbol{\rho})e^{-i\mathbf{q} \cdot \boldsymbol{\rho}}$, from the convolution theorem, the momentum wavefunction

$$\tilde{\psi}(\mathbf{q}_1, \mathbf{q}_2) = [\tilde{U}(\mathbf{q}_1)\tilde{U}(\mathbf{q}_2)] * \left[\tilde{c}\left(\frac{\mathbf{q}_1 - \mathbf{q}_2}{2}\right)\delta(\mathbf{q}_1 + \mathbf{q}_2) \right] \quad (3)$$

where “*” is the convolution sign. In classical SHWS, $|\tilde{U}(\mathbf{q})|^2$ is the measured intensity from the field $U(\boldsymbol{\rho})$, and the average $\mathbf{q}$ is the average phase gradient in this aperture¹³

$$\langle \mathbf{q} \rangle = \frac{1}{(2a)^2} \int_S d\boldsymbol{\rho} \nabla \Phi(\boldsymbol{\rho}) = \langle \nabla \Phi(\boldsymbol{\rho}) \rangle_S \quad (4)$$

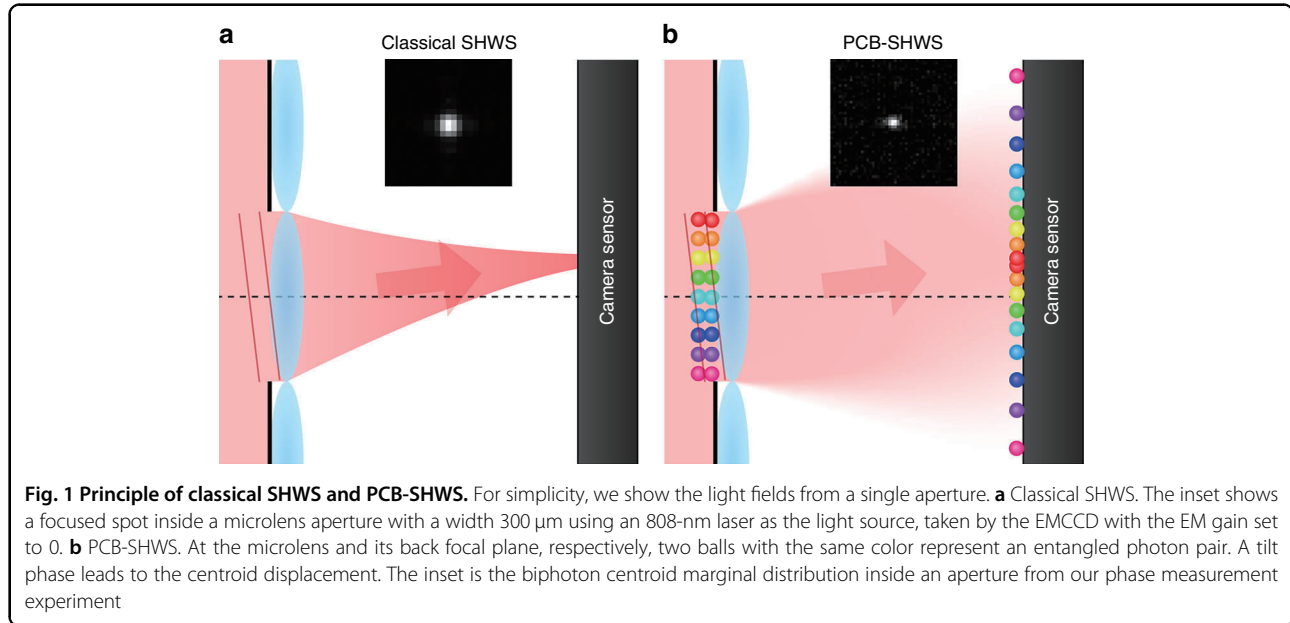
For $\Phi(\boldsymbol{\rho})$ whose gradient is slow-varying within the aperture (in order to satisfy the spatial resolution limit), $\tilde{U}(\mathbf{q})$ is still peaked at $\langle \mathbf{q} \rangle$, although the exact distribution is a bit different from the sinc function in the plane-wave case. From Eq. (3), considering the displacement and broadening effects of convolution, their momenta are approximately anti-correlated with the center $\langle \mathbf{q} \rangle$, so the measured centroid distribution is peaked at $\langle \mathbf{q} \rangle$. The spot in the direct image is a slightly blurred $|\tilde{c}(\mathbf{q})|^2$, which may be larger than an aperture (see Supplementary Information 2 for the experimental images), as the biphoton field is spatially incoherent in the first order. So, the phase added to such a biphoton field cannot be measured by classical SHWS.

Under the approximation of a perfect position correlation $c(\boldsymbol{\rho}) \approx \delta(\boldsymbol{\rho})$, Eq. (3) can be further simplified to

$$\begin{aligned} \tilde{\psi}(\mathbf{q}_1, \mathbf{q}_2) &\approx [\tilde{U}(\mathbf{q}_1)\tilde{U}(\mathbf{q}_2)] * \delta(\mathbf{q}_1 + \mathbf{q}_2) \\ &= \int d\mathbf{q} \tilde{U}(\mathbf{q})\tilde{U}(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}) = \tilde{U}_2(\mathbf{q}_1 + \mathbf{q}_2) \end{aligned} \quad (5)$$

where $\tilde{U}_2(\mathbf{q})$ is the Fourier transform of $[U(\boldsymbol{\rho})]^2$ whose phase is $2\Phi(\boldsymbol{\rho})$. The centroid distribution $\Gamma_c(\mathbf{q}_c) =$





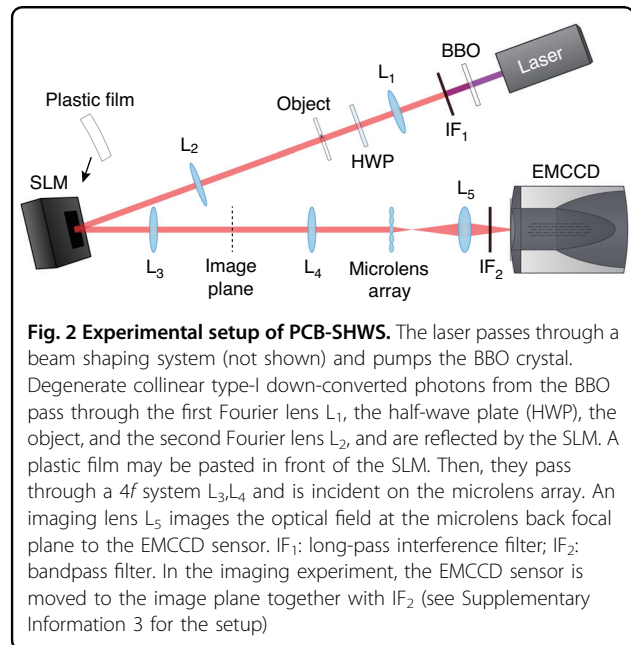
$|\tilde{U}_2(2\mathbf{q}_c)|^2$, so

$$\langle \mathbf{q}_c \rangle = \frac{1}{2} \frac{1}{(2a)^2} \int_S d\mathbf{p} \nabla [2\Phi(\mathbf{p})] = \langle \nabla \Phi(\mathbf{p}) \rangle_S \quad (6)$$

exactly corresponds to the average phase gradient in classical SHWS, while the peak width is half the classical result. Experimentally, the centroid distribution can be efficiently measured³¹ and has a twice-pixel resolution as direct images³². In spite of these differences, the data processing methods of classical SHWS can be applied to the centroid distribution for phase reconstruction. The principle of classical SHWS and PCB-SHWS is shown in Fig. 1, where the insets are a direct image of classical SHWS and a centroid distribution of PCB-SHWS inside an aperture.

Experimental setup

We use an electron-multiplying charge-coupled device (EMCCD) camera and the multiple frame method developed by Defienne et al.^{33–35} to measure the biphoton JPD^{8,9,11,21–23,30,32,36,37}. The setup of the phase measurement experiment is shown in Fig. 2. A horizontally-polarized (H) laser beam at 404 nm pumps a β -barium borate (BBO) crystal with the thickness 1 mm and is removed by a long-pass interference filter (IF). Degenerate collinear type-I down-converted photon pairs at the vertical (V) polarization pass through the first Fourier lens ($f_1 = 15$ cm) and are switched to H polarization by a half-wave plate. Their positions are anti-correlated at their focal plane, where the object (USAF 1951 resolution target) is placed at one-half of the beam. After the second Fourier lens ($f_2 = 25$ cm), an SLM adds the same phase pattern to each photon. Then, they pass through a 4f

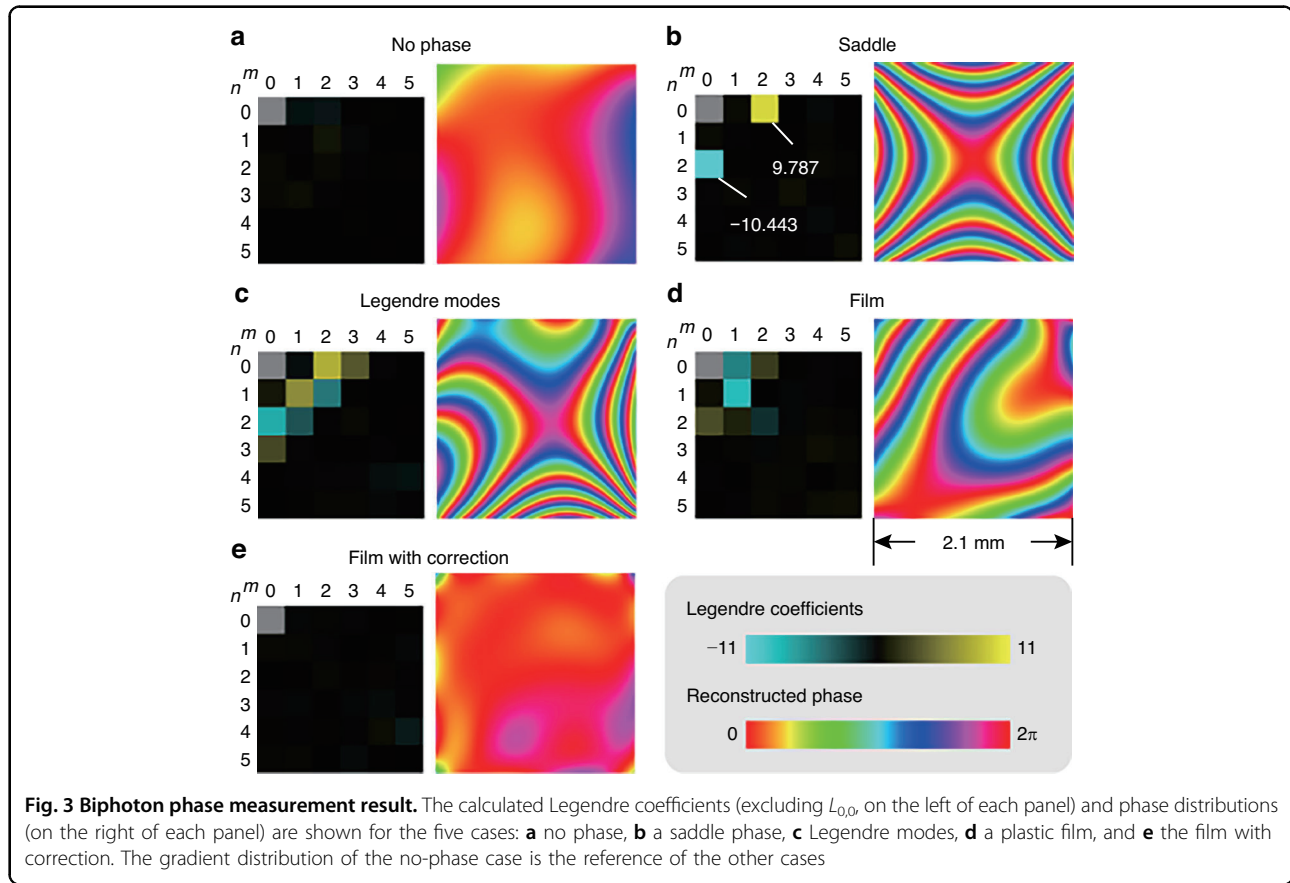


system ($f_3 = f_4 = 15$ cm) and arrive at the microlens array with the aperture width 300 μm and focal length $f_{\text{SH}} = 14.6$ mm. An imaging lens ($f_5 = 5$ cm) projects the optical field at the microlens back focal plane to the EMCCD sensor with the magnification ratio -1 , and a bandpass IF selects near-degenerate down-converted photons. See Materials and methods for details.

Biphoton phase measurement

As the region of interest (ROI) in our experiment is a square, we use the modal method based on two-dimensional





Legendre polynomials³⁸ $L_{m,n}(\rho) = L_m(x)L_n(y)$ to reconstruct the phase distribution from the gradient, rather than Zernike polynomials which are suitable for circular ROIs. See Materials and methods and Supplementary Information 1 for data processing algorithms.

We measure the phase of five cases: no phase added, a hyperbolic paraboloid (saddle) phase $10(L_{2,0} - L_{0,2})$, a superposition of several Legendre modes

$$8L_{2,0} + 6L_{1,1} - 7L_{0,2} + 4L_{3,0} - 5L_{2,1} - 4L_{1,2} + 3L_{0,3} \quad (7)$$

a plastic film placed in front of the SLM, and the plastic film with phase correction. The exposure times, direct images, centroid distributions, and gradient distributions are shown in Supplementary Information 2. From the direct images, the object pattern is faintly visible in each aperture because it modifies $\tilde{c}(\mathbf{q})$. The measured gradient distribution of the no-phase case still deviates from zero, which is mainly due to the aberration of the imaging lens, so it serves as a reference by which the measured gradient distributions of other cases are subtracted. The calculated Legendre coefficients (see the tables in Supplementary Information for values) and the reconstructed phase distributions are shown in Fig. 3. In the saddle phase case,

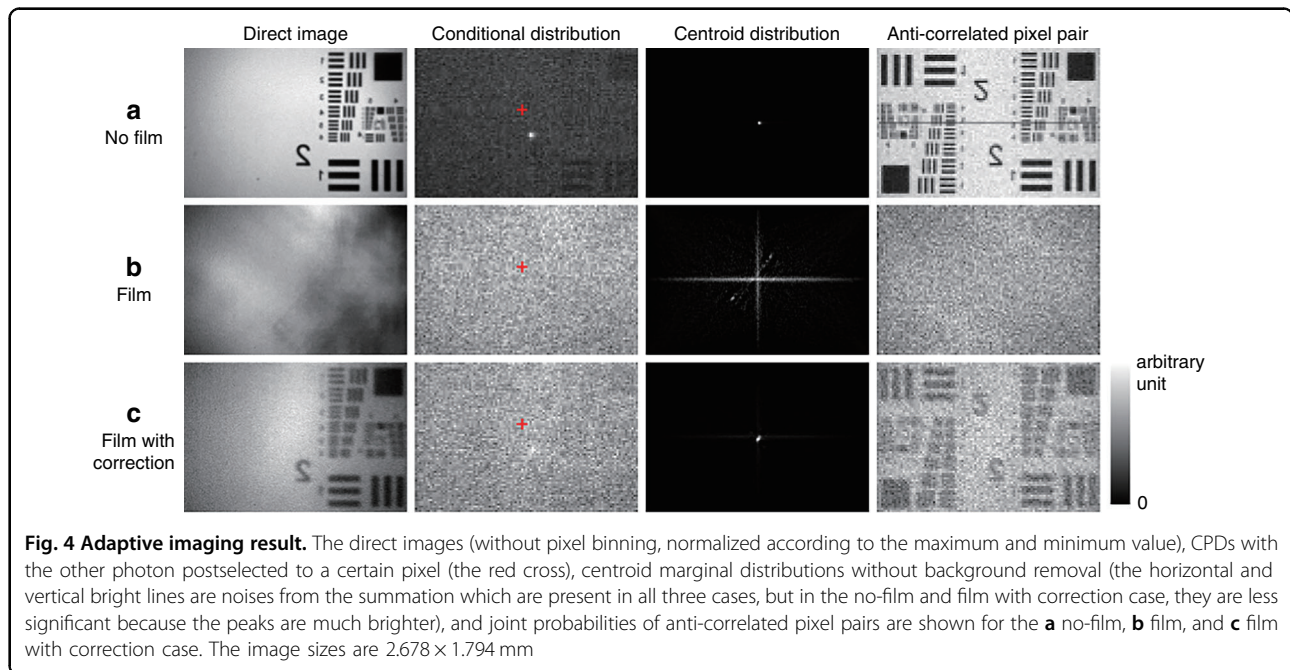
the calculated $L_{2,0}$ and $L_{0,2}$ are close to the theoretical values 10 and -10 , respectively. In the cases of Legendre modes and film with correction, ignoring the tilt terms $L_{1,0}$ and $L_{0,1}$, which do not blur the image, compared to Eq. (7) and 0, respectively, the root mean square error values of the phase distributions (discretized into 120×120 pixels; in the unit of optical path difference) are 0.0623λ and 0.0502λ . So, the measured phases fit well with the designed ones on the SLM, and the aberration caused by the film is approximately corrected. The errors are mainly from the imperfection of SLM, slight misalignment of the setup, image distortion from the imaging lens, and noise from the multiple frame method.

Adaptive imaging

To test whether the biphoton phase is correctly canceled after PCB-SHWS, we then perform the adaptive imaging experiment. The EMCCD sensor is moved to the back focal plane of the third Fourier lens together with the bandpass IF. See Supplementary Information 3 for a figure of this new setup and a brief principle of biphoton imaging with aberrations. When taking frames, 2×2 pixels are binned into one on the EMCCD to increase the frame rate⁸.

We take 4.41×10^6 frames in each of the three cases: no film, film, and film with correction, as well as direct





images without pixel binning. The direct images, conditional probability distributions (CPDs) of one photon with the other photon postselected to a certain pixel, centroid distributions, and JPDs of anti-correlated pixel pairs are shown in Fig. 4. The anti-correlated pair distributions have central symmetry, so the object patterns exist on the other halves of the images. When the film is present, the object cannot be identified in the direct image or the anti-correlated pair distribution. After correction, they are visible again, but a bit blurry because the slightly curved film about 2 mm away from the SLM liquid crystal plate cannot be regarded as a pure phase object at exactly the SLM plane. A part of the photons are reflected or scattered after passing through the film twice, equivalent to a lower quantum efficiency of the sensor, so the signal-to-noise ratios (SNRs) of the conditional distribution and the anti-correlated pair distribution in the film with correction case are much lower than the no-film case. The centroid distribution, which is the metric of the sensorless adaptive imaging method¹¹, is peaked at the center point in the no-film or film with correction case, while scattered in the film case.

Discussion

In this work, we introduced PCB-SHWS to measure the phase pattern added on biphotons with a strong position correlation, and performed experiments of Legendre coefficient measurement and adaptive imaging against the disturbance of a plastic film. Both PCB-SHWS and the sensorless approach¹¹ do not require classical detection of the aberration source^{24,25}, reference beams¹⁹, or

polarization entanglement^{21,22}, while our method only requires one measurement step and has the advantage in efficiency. Compared to the multiple frame method, with a more advanced biphoton JPD measurement technique like the time-stamping camera^{20,39}, the phase can be rapidly acquired for real-time (truly adaptive) correction of a time-varying aberration, suitable for future quantum communication and imaging research. However, limited by the microlens width and camera pixel width which affect the spatial resolution and sensitivity, respectively, like classical SHWS, the result of PCB-SHWS may not be precise enough. So, in order to restore a quantum image from common aberrations perfectly in quantum microscopy, PCB-SHWS can first provide a good estimate of the phase, and then mode coefficient scanning can precisely determine it. By reducing the microlens size or magnifying the beam, the spatial resolution can be improved to some extent (with the sensitivity reduced), but it is, after all, not applicable to extremely detailed phase patterns such as those from diffusers^{23–25}.

If the photons are anti-correlated in position, the effective phase is an even function³⁵, and its measurement scheme can also be analyzed, but the lateral position of the microlens array should be adjusted so that the anti-correlation center is the center, a side midpoint, or a vertex of an aperture, as detailed in Supplementary Information 4.

Although the essential sensing setup (a microlens array and a camera capable of JPD measurement) in PCB-SHWS also appears in the method named quantum SHWS from our previous work³⁰, their theoretical frameworks, data processing methods, and scopes of



application are distinct, and a biphoton state suitable for one method is unsuitable for the other. A clarification of their differences is given in Supplementary Information 5. Also, in classical optics, apart from using only the peak positions for SHWS, the whole momentum distribution from each microlens aperture provides more information about the optical field^{40,41}, which has been incorporated in a new technique, namely plenoptic imaging or light-field imaging^{42,43}. Therefore, the microlens array, which enables position and local momentum measurement without breaking the uncertainty principle, has great potential in detecting higher-order correlation properties of multiphoton optical fields when combined with efficient photon coincidence detection techniques. Its further applications in quantum optics can be explored in the future.

Materials and methods

Details of the experimental setup

Before pumping the BBO, the continuous-wave laser beam (TOPTICA) passes through two Fourier lenses for magnification, two cylindrical lenses to adjust its shape, and a short-pass filter at 600 nm. The long-pass IF is at 647 nm. The SLM (Hamamatsu X13267) has 792×600 pixels whose widths are $12.5 \mu\text{m}$. We choose $f_2 > f_3$ because the beam at the back focal plane of the first Fourier lens is too large for the EMCCD (Andor iXon Ultra 888; pixel width: $13 \mu\text{m}$) in the imaging setup, and thus the image of the object is shrunk (we focus on proof-of-principle aberration cancellation in this work and do not discuss quantum superresolution imaging or image distillation). The imaging lens is used because the distance between the EMCCD casing and its sensor is 1.75 cm , which is larger than f_{SH} , and it is not easy to put the microlens array (LBTEK) inside the casing (a shutter is at its inner side). This will introduce distortions when imaging, so the beam should be at the center of the imaging lens and not too large. The center wavelength of the bandpass IF $[(810 \pm 5) \text{ nm}]$ is not 808 nm , but degenerate photon pairs can pass through it, and the only shortcoming is the existence of some single-photon incidences, which are treated as dark counts.

Using the formulae in ref.⁴⁴, the full width at half maximum (FWHM) of the biphoton CPD (the two-dimensional probability distribution of one photon when the other is postselected to a given position) is about $17 \mu\text{m}$ at the BBO and $28 \mu\text{m}$ at the microlens array, so most of the photon pairs pass through the same microlens; the FWHM of the spot at the back focal plane from one aperture is about $604 \mu\text{m}$, larger than the aperture.

The ROI of the EMCCD is set to 165×165 pixels in the phase measurement experiment and 105×71 (binned) in the imaging one. The EM gain (set to 0 when taking the direct image in Fig. 1a), horizontal pixel readout rate,

vertical pixel shift speed, and vertical clock voltage amplitude are set to 1000, 10 MHz, $0.6 \mu\text{s}$ and $+2 \text{ V}$, respectively. The exposure times are different in each measurement, determined by the beam intensity, so that roughly $1/5$ of the pixels in the ROI have photons in each frame³⁴ (see Supplementary Information 2 for the values in the phase measurement experiment. In the imaging experiment, the exposure times are 1.4 ms, 1.8 ms, and 2.4 ms for the no-film, film, and film with correction case, respectively. The EMCCD sensor is cooled to -53°C and -38°C in the phase measurement and imaging (a higher frame rate produces more heat) experiment, respectively. When taking the three direct images, the exposure time is 0.1 s, and 1000 frames are directly summed for each case.

Data processing

In the multiple frame method, the EMCCD captures N frames with grayscale values from 0 to 65535, and a threshold binarizes the frame data. In our experiment, by evaluating the grayscale value histogram without the down-converted photons, the threshold is set to 509. Denoting the counting of the i th pixel (the indexing is arbitrary) of the n th frame as $C_{n,i}$ ($C_{n,i} = 0$ or 1), defining the single-pixel count average $\langle C_i \rangle = \frac{1}{N} \sum_{n=1}^N C_{n,i}$ and the two-pixel coincidence average $\langle C_{ij} \rangle = \frac{1}{N} \sum_{n=1}^N C_{n,i} C_{n,j}$, if the Poissonian statistics of photons is assumed, the JPD is estimated by the covariance of the counts of two pixels $\Gamma_{ij} \approx \langle C_{ij} \rangle - \langle C_i \rangle \langle C_j \rangle$. Its derivation has taken the quantum efficiency and the dark count into account³⁴. The SNR is a major issue, so it is only suitable for biphoton states with narrow CPDs³⁶. The JPD of the same pixel cannot be measured by this method, and JPD of two pixels near each other on the same line has abnormal correlations due to the charge smearing effect of EMCCD⁷, so linear interpolation is applied when necessary: for two pixels on the same line (x_1, y) and (x_2, y) , if $|x_1 - x_2| \leq 10$ pixels (no such condition in the imaging experiment), their JPD value $\Gamma_{(x_1, y), (x_2, y)}$ are replaced by $(\Gamma_{(x_1, y), (x_2, y+1)} + \Gamma_{(x_1, y), (x_2, y-1)})/2$. If the photons have a strong position anti-correlation, the interpolated result will be lower than normal, as shown in the no-film case in Fig. 4a.

The actual count rate drifts over time, adding a positive background to the calculated Γ_{ij} ³³, so some works use a successive frame formula^{9,21} or discard frames with too few or too many pixels with counting³⁰ to reduce it, while the SNR is reduced⁸. In PCB-SHWS, we only need the peak positions in the centroid distribution, rather than actual values which are influenced by the background, so we use the original covariance formula to calculate the JPD. Then, the centroid distribution is calculated from the JPD, and the peak positions are extracted using an algorithm and converted to gradient values. See Supplementary Information 1 for details of data processing, including



the algorithms and a discussion about the SNR and the number of frames.

Phase reconstruction

The two-dimensional Legendre polynomials³⁸ $L_{m,n}(\rho) = L_m(x)L_n(y)$, where

$$L_l(x) = \frac{1}{2^l} \sum_{k=0}^{\lfloor l/2 \rfloor} \frac{(-1)^k (2l-2k)!}{k!(l-k)!(l-2k)!} x^{l-2k} \quad (8)$$

is defined on $-1 \leq x \leq 1$. The gradient values are multiplied by 1.05 mm to rescale the ROI from 2.1×2.1 mm (7×7 apertures) to 2×2 with the dimension removed. We choose the maximum m and n to be 5 (higher modes are rapidly oscillating), so there are 35 modes excluding the constant phase mode $L_{0,0} = 1$. For an aperture centered at (x_0, y_0) after rescaling, denoting $L_l|_a^b = L_l(b) - L_l(a)$ and $\Lambda_l|_a^b = \int_a^b dx L_l(x)$, the equations are

$$\begin{aligned} \kappa_x &= \frac{1}{4a^2} \sum_{m,n} \alpha_{m,n} L_m|_{x_0-a}^{x_0+a} \Lambda_n|_{y_0-a}^{y_0+a} \\ \kappa_y &= \frac{1}{4a^2} \sum_{m,n} \alpha_{m,n} \Lambda_m|_{x_0-a}^{x_0+a} L_n|_{y_0-a}^{y_0+a} \end{aligned} \quad (9)$$

where $2a$ is the aperture width after rescaling ($2/7$ in our experiment). Considering all 49 apertures, we have 98 equations. The coefficients $\alpha_{m,n}$ are solved by the least square method, and the phase is reconstructed $\Phi(\rho) = \sum_{m,n} \alpha_{m,n} L_{m,n}(\rho)$.

Acknowledgements

This work was funded by the Innovation Program for Quantum Science and Technology (Grant Nos. 2021ZD0301200 and 2021ZD0301400), the National Natural Science Foundation of China (Grant Nos. 92365205, 11821404, and W2411001), and the USTC Major Frontier Research Program (Grant No. LS2030000002).

Author details

¹Laboratory of Quantum Information, University of Science and Technology of China, 230026 Hefei, China. ²Anhui Province Key Laboratory of Quantum Network, University of Science and Technology of China, 230026 Hefei, China. ³CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, 230026 Hefei, China. ⁴Hefei National Laboratory, University of Science and Technology of China, 230088 Hefei, China

Author contributions

Y.Z. conceived the idea, performed the experiments, and analyzed the data. Z.-D.L. and J.-S.T. provided assistance on data acquisition and processing. J.-S.X., C.-F.L., and G.-C.G. supervised the project. All authors participated in the discussion and analysis of the manuscript.

Data availability

All data needed to evaluate the conclusions in this paper are present in the paper and/or the Supplementary Information. Additional data, including the raw camera frames, is available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare no competing interests.

Supplementary information The online version contains supplementary material available at <https://doi.org/10.1038/s41377-025-02024-4>.

Received: 11 May 2025 Revised: 22 July 2025 Accepted: 22 August 2025
Published online: 08 September 2025

References

- Zhang, Z. S. et al. Entanglement-based quantum information technology: a tutorial. *Adv. Opt. Photonics* **16**, 60–162 (2024).
- Moreau, P. A. et al. Imaging with quantum states of light. *Nat. Rev. Phys.* **1**, 367–380 (2019).
- Defienne, H. et al. Advances in quantum imaging. *Nat. Photonics* **18**, 1024–1036 (2024).
- Pittman, T. B. et al. Optical imaging by means of two-photon quantum entanglement. *Phys. Rev. A* **52**, R3429–R3432 (1995).
- Lemos, G. B. et al. Quantum imaging with undetected photons. *Nature* **512**, 409–412 (2014).
- Giovannetti, V. et al. Sub-rayleigh-diffraction-bound quantum imaging. *Phys. Rev. A* **79**, 013827 (2009).
- Toninelli, E. et al. Resolution-enhanced quantum imaging by centroid estimation of biphotons. *Optica* **6**, 347–353 (2019).
- He, Z. et al. Quantum microscopy of cells at the Heisenberg limit. *Nat. Commun.* **14**, 2441 (2023).
- Defienne, H. et al. Quantum image distillation. *Sci. Adv.* **5**, eaax0307 (2019).
- Gregory, T. et al. Imaging through noise with quantum illumination. *Sci. Adv.* **6**, eaay2652 (2020).
- Cameron, P. et al. Adaptive optical imaging with entangled photons. *Science* **383**, 1142–1148 (2024).
- Platt, B. C. & Shack, R. History and principles of Shack-Hartmann wavefront sensing. *J. Refract. Surg.* **17**, S573–S577 (2001).
- Ares, J., Mancebo, T. & Bará, S. Position and displacement sensing with Shack-Hartmann wave-front sensors. *Appl. Opt.* **39**, 1511–1520 (2000).
- Zheng, Y. et al. Detecting momentum weak value: Shack-Hartmann versus a weak measurement wavefront sensor. *Opt. Lett.* **46**, 5352–5355 (2021).
- Southwell, W. H. Wave-front estimation from wave-front slope measurements. *J. Opt. Soc. Am.* **70**, 998–1006 (1980).
- Ragazzoni, R. Pupil plane wavefront sensing with an oscillating prism. *J. Mod. Opt.* **43**, 289–293 (1996).
- Yang, M. et al. Zonal reconstruction of photonic wavefunction via momentum weak measurement. *Laser Photonics Rev.* **14**, 1900251 (2020).
- Booth, M. J. Wave front sensor-less adaptive optics: a model-based approach using sphere packings. *Opt. Express* **14**, 1339–1352 (2006).
- Black, A. N. et al. Quantum-enhanced phase imaging without coincidence counting. *Optica* **10**, 952–958 (2023).
- Zia, D. et al. Interferometric imaging of amplitude and phase of spatial biphoton states. *Nat. Photonics* **17**, 1009–1016 (2023).
- Defienne, H. et al. Polarization-entanglement-enabled quantum holography. *Nat. Phys.* **17**, 591–597 (2021).
- Camphausen, R. et al. A quantum-enhanced wide-field phase imager. *Sci. Adv.* **7**, eabj2155 (2021).
- Defienne, H., Reichert, M. & Fleischer, J. W. Adaptive quantum optics with spatially entangled photon pairs. *Phys. Rev. Lett.* **121**, 233601 (2018).
- Lib, O., Hasson, G. & Bromberg, Y. Real-time shaping of entangled photons by classical control and feedback. *Sci. Adv.* **6**, eabb6298 (2020).
- Shekel, R., Lib, O. & Bromberg, Y. Shaping entangled photons through arbitrary scattering media using an advanced wave beacon. *Opt. Quantum* **2**, 303–309 (2024).
- Walborn, S. P. et al. Spatial correlations in parametric down-conversion. *Phys. Rep.* **495**, 87–139 (2010).
- Schneeloch, J. & Howell, J. C. Introduction to the transverse spatial correlations in spontaneous parametric down-conversion through the biphoton birth zone. *J. Opt.* **18**, 053501 (2016).
- Einstein, A., Podolsky, B. & Rosen, N. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev. A* **47**, 777–780 (1935).
- Zheng, Y. et al. Reconstructing the multiphoton spatial wave function with coincidence wave-front sensing. *Phys. Rev. A* **107**, 042608 (2023).



30. Zheng, Y. et al. Characterizing biphoton spatial wave function dynamics with quantum wavefront sensing. *Phys. Rev. Lett.* **133**, 033602 (2024).
31. Tsang, M. Quantum imaging beyond the diffraction limit by optical centroid measurements. *Phys. Rev. Lett.* **102**, 253601 (2009).
32. Defienne, H. et al. Pixel super-resolution with spatially entangled photons. *Nat. Commun.* **13**, 3566 (2022).
33. Defienne, H., Reichert, M. & Fleischer, J. W. General model of photon-pair detection with an image sensor. *Phys. Rev. Lett.* **120**, 203604 (2018).
34. Reichert, M., Defienne, H. & Fleischer, J. W. Optimizing the signal-to-noise ratio of biphoton distribution measurements. *Phys. Rev. A* **98**, 013841 (2018).
35. Cameron, P. et al. Shaping the spatial correlations of entangled photon pairs. *J. Phys. Photonics* **6**, 033001 (2024).
36. Bhattacharjee, A. et al. Propagation-induced revival of entanglement in the angle-OAM bases. *Sci. Adv.* **8**, eabn7876 (2022).
37. Kam, A. et al. Near-field photon entanglement in total angular momentum. *Nature* **640**, 634–640 (2025).
38. Niu, K. & Tian, C. Zernike polynomials and their applications. *J. Opt.* **24**, 123001 (2022).
39. Courme, B. et al. Quantifying high-dimensional spatial entanglement with a single-photon-sensitive time-stamping camera. *Opt. Lett.* **48**, 3439–3442 (2023).
40. Hradil, Z., Řeháček, J. & Sánchez-Soto, L. L. Quantum reconstruction of the mutual coherence function. *Phys. Rev. Lett.* **105**, 010401 (2010).
41. Stoklasa, B. et al. Wavefront sensing reveals optical coherence. *Nat. Commun.* **5**, 3275 (2014).
42. D'Angelo, M. et al. Correlation plenoptic imaging. *Phys. Rev. Lett.* **116**, 223602 (2016).
43. Paniate, A. et al. Light-field ghost imaging. *Phys. Rev. Appl.* **21**, 024032 (2024).
44. Zheng, Y. et al. Theory of the monochromatic advanced-wave picture and applications in biphoton optics. *Phys. Rev. A* **110**, 063710 (2024).

